Optimal Transmission Strategy without Capacity Loss at a Primary User in Cognitive Radio Networks over Inter-Symbol Interference Channels

Woohyuk Chang, *Member, IEEE,* and Bang Chul Jung, *Member, IEEE*

Abstract—In this letter, we consider a cognitive radio (CR) communication over inter-symbol interference channels. We propose a transmission strategy for a secondary sender to amplify the received signal from a primary sender and to transmit it together with its own signal by using characteristics of ISI channels. It is shown that the proposed strategy makes it possible for the secondary network to achieve considerable throughput without causing any capacity loss at the primary network, and it outperforms the convetional CR scheme.

Index Terms—Cognitive radio networks, inter-symbol interference, power control, water-filling, amplify-and-forward relaying.

I. Introduction

COGNITIVE radios (CRs) have been recognized as one of the most promising technologies to overcome lack of wireless spectrum bands by allowing unlicensed users to send their data over licensed users' bands which are temporarily unused [1]. In [2], achievable rate regions of CR networks are derived when the primary user's data are known at the secondary sender in a noncausal manner by a genie or in a causal manner by decoding the received signals from the primary sender. It is, however, assumed in [2] that both primary sender and receiver have a prior information on the secondary sender and receiver, which may not be feasible in practice. When the primary receiver has the codebook of the secondary sender, the secondary sender can be allowed to transmit its signal if the primary receiver can first decode the signal from the secondary sender by treating the signal from the primary sender as interference, and then decode the signal from the primary sender after subtracting the signal from the secondary sender [3]. This technique is called *successive interference cancellation (SIC)*. The SIC also requires the primary receiver to have a prior information on the secondary network. Another transmission technique allows simultaneous transmissions of primary and secondary senders as long as the secondary sender interferes less than a certain threshold, called *interference-temperature*, with the primary receiver [4]. It, however, sacrifices the primary channel capacity.

In this letter, we raise a fundamental question on CR communications. Assuming that the secondary sender and receiver are aware of the primary channels and of how the primary sender and receiver operate, is it possible for the secondary sender to transmit data without causing capacity loss at the primary network, while the primary sender and receiver still operate even without any prior knowledge on the secondary sender and receiver? In general, it is hard to say *yes* for this question. Nevertheless, we give a positive answer for the case of CR communications over inter-symbol interference (ISI) channels. Instead of decoding the primary user's data at the secondary sender, the secondary sender simply amplifies the received signal from the primary sender and transmit them together with its own signal in the proposed strategy. Although the primary sender and receiver operate as if the secondary nodes do not exist, we show that the secondary sender can transmit data without causing any capacity loss at the primary network if the primary channel is given as an ISI channel.

II. Primary Channel Capacity

In this section, we first consider the primary ISI channel as shown in Fig. 1, where it is assumed that there exist no secondary sender and receiver. The primary sender S_1 transmits a complex-valued codeword $\underline{X}_1 = (X_1(1), \cdots, X_1(T))$ with an average power constraint of P_1 to the primary receiver D_1 through an ISI channel with its channel tap values $\underline{h}_{11} = (h_{11,0}, \cdots, h_{11,D_{11}})$, where D_{11} denotes the maximum number of taps of delay spread ($D_{11} > 0$). Then, the received signal vector $\underline{Y}_1 = (Y_1(1), \cdots, Y_1(T))$ at D_1 is given by

$$Y_1(t) = \sum_{i=0}^{D_{11}} h_{11,i} X_1(t-i) + Z_1(t), \quad t = 1, \cdots, T, \quad (1)$$

where $\underline{Z}_1 = (Z_1(1), \cdots, Z_1(T))$ is a noise vector consisting of T zero-mean, i.i.d. circularly symmetric complex Gaussian random variables with a variance σ_1^2. The capacity of the primary ISI channel is obtained using the Fourier transform (FT) at D_1 which decomposes the channel into independent parallel memoryless channels.[1] Assuming that $\underline{h}_{11}$ is known at S_1, the primary ISI channel capacity can be written as [5]:

$$C_1 = \max_{\Sigma_1(w)} \frac{1}{\pi} \int_0^\pi C\left(\frac{\Sigma_1(w)|H_{11}(w)|^2}{\sigma_1^2}\right) dw \quad (2a)$$

$$\text{subject to } \Sigma_1(w) \geq 0 \text{ for all } w \quad (2b)$$

$$\frac{1}{\pi} \int_0^\pi \Sigma_1(w) dw \leq P_1, \quad (2c)$$

where $C(x) \triangleq \log_2(1+x)$ [bits/sec/Hz], $\Sigma_1(w)$ is the transmit power over frequency-domain w, and $H_{11}(w)$ is the FT of $\underline{h}_{11}$. As a result, the optimal power allocation $\Sigma_1^*(w)$ over

[1] Together with the FT at D_1, the inverse FT can be used at S_1 after encoding data into a codeword. In this case, $\underline{X}_1 = (X_1(1), \cdots, X_1(T))$ is the output of the inverse FT.
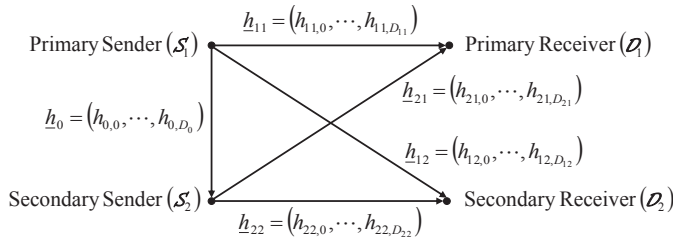
Fig. 1.   Channel model.

frequency-domain w is given by the well-known water-filling solution [5] such as

$$\Sigma_1^*(w) = \left[\frac{1}{\lambda_1 \ln 2} - \frac{\sigma_1^2}{|H_{11}(w)|^2} \right]^+, \qquad (3)$$

where λ_1 denotes Lagrange multiplier associated with the constraint (2c).

III. ACHIEVABLE RATES FOR SECONDARY CHANNEL

We now consider both the primary and the secondary ISI channels together as shown in Fig. 1. The transmitted codeword $\underline{X}_1$ at $\mathcal{S}_1$ is also received at the secondary sender $\mathcal{S}_2$ and at the secondary receiver $\mathcal{D}_2$ through ISI channels with their channel tap values $\underline{h}_0 = (h_{0,0}, \cdots, h_{0,D_0})$ and $\underline{h}_{12} = (h_{12,0}, \cdots, h_{12,D_{12}})$, respectively. Similarly, the $\mathcal{S}_2$ transmits a complex-valued codeword $\underline{X}_2 = (X_2(1), \cdots, X_2(T))$ with an average power constraint of P_2, and it is received at $\mathcal{D}_1$ and $\mathcal{D}_2$ through ISI channels with their channel tap values $\underline{h}_{21} = (h_{21,0}, \cdots, h_{21,D_{21}})$ and $\underline{h}_{22} = (h_{22,0}, \cdots, h_{22,D_{22}})$, respectively. Then, the received signal vectors at $\mathcal{S}_2$, $\mathcal{D}_1$, and $\mathcal{D}_2$ are given by

$$Y_0(t) = \sum_{i=0}^{D_0} h_{0,i} X_1(t-i) + Z_0(t), \qquad (4)$$

$$Y_1(t) = \sum_{i=0}^{D_{11}} h_{11,i} X_1(t-i) + \sum_{i=0}^{D_{21}} h_{21,i} X_2(t-i) + Z_1(t), (5)$$

$$Y_2(t) = \sum_{i=0}^{D_{22}} h_{22,i} X_2(t-i) + \sum_{i=0}^{D_{12}} h_{12,i} X_1(t-i) + Z_2(t), (6)$$

$t = 1, \cdots, T$, respectively, where $\underline{Z}_i = (Z_i(1), \cdots, Z_i(T))$, $i \in \{0, 1, 2\}$ are noise vectors consisting of T zero-mean, i.i.d. circularly symmetric complex Gaussian random variables with variances σ_i^2, $i \in \{0, 1, 2\}$, respectively. Assuming that $\underline{h}_0$, $\underline{h}_{11}$, $\underline{h}_{12}$, $\underline{h}_{21}$ and $\underline{h}_{22}$ are known at $\mathcal{S}_2$, we consider the following two achievable schemes for the secondary nodes.

A. Conventional Scheme

We first consider a simple conventional scheme where $\mathcal{S}_2$ transmits data only through the frequency bands which are not used by the primary nodes [6].[2] Hence, an achievable rate for

[2]In [6], this conventional scheme is called as the conventional CR system (C-CR1/2) with interference power limit set to zero ($Q = 0$).

the secondary channel is given by

$$R_{2,1} = \max_{\Sigma_2(w)} \frac{1}{\pi} \int_0^\pi C\left(\frac{\Sigma_2(w)|H_{22}(w)|^2}{\sigma_2^2} \right) dw \qquad (7a)$$

subject to $\Sigma_2(w) \geq 0$ for all w \qquad (7b)

$$\frac{1}{\pi} \int_0^\pi \Sigma_2(w) dw \leq P_2, \qquad (7c)$$

$\Sigma_2(w) = 0$ for all w satisfying $\Sigma_1^*(w) > 0$, (7d)

where $\Sigma_2(w)$ is the transmit power over frequency-domain w and $H_{22}(w)$ is the FT of $\underline{h}_{22}$. Then, the optimal power allocation $\Sigma_2^*(w)$ is given by

$$\Sigma_2^*(w) = \begin{cases} 0, & \text{for } \Sigma_1^*(w) > 0 \\ \left[\frac{1}{\lambda_{2,1} \ln 2} - \frac{\sigma_2^2}{|H_{22}(w)|^2} \right]^+, & \text{for } \Sigma_1^*(w) = 0, \end{cases} \qquad (8)$$

where $\lambda_{2,1}$ denotes Lagrange multiplier associated with the constraint (7c).

This conventional scheme works well when the primary nodes operate in the low SNR regime since lots of frequency bands are not used by the primary nodes. On the other hand, when the primary nodes operate in the high SNR regime, there is little chance for the secondary sender to transmit data since most frequency bands are already used by the primary nodes.

B. Proposed Scheme

To enhance the performance of the conventional scheme, we propose a superimposed amplify-and-forward (AF) relaying scheme for the secondary sender $\mathcal{S}_2$. Dividing the available average transmit power P_2 at $\mathcal{S}_2$ into αP_2 and $(1 - \alpha)P_2$, $\alpha \in [0, 1]$, the received signal $\underline{Y}_0 = (Y_0(1), \cdots, Y_0(T))$ from $\mathcal{S}_1$ is amplified so that its transmit average power becomes αP_2 while $\mathcal{S}_2$'s own data are encoded into signal $\underline{X}_{21} = (X_{21}(1), \cdots, X_{21}(T))$ with its average transmit power of $(1 - \alpha)P_2$, and then these two signals are summed and transmitted. We assume that this processing time is negligibly small.[3] Therefore, the transmitted signal $\underline{X}_2 = (X_2(0), \cdots, X_2(T))$ at $\mathcal{S}_2$ is given by

$$X_2(t) = \Gamma Y_0(t) + X_{21}(t) = \Gamma \sum_{j=0}^{D_0} h_{0,i} X_1(t-i) + X_{21}(t)$$
$$+ \Gamma Z_0(t), \quad t = 1, \cdots, T, \qquad (9)$$

where $\Gamma \triangleq \frac{\sqrt{\alpha P_2}}{\sqrt{\sigma_0^2 + \sum_{j=0}^{D_0} |h_{0,j}|^2 P_1}}$. From (5) and (9), the received signal at $\mathcal{D}_1$ is then given by

$$Y_1(t) = \sum_{i=0}^{D_{11}} h_{11,i} X_1(t-i) + \sum_{i=0}^{D_{21}} h_{21,i} \left\{ \Gamma \sum_{j=0}^{D_0} h_{0,j} X_1(t-i-j) \right.$$
$$\left. + X_{21}(t-i) + \Gamma Z_0(t-i) \right\} + Z_1(t)$$

$$= \sum_{i=0}^{D_{11}} h_{11,i} X_1(t-i) + \Gamma \sum_{i=0}^{D_{21}} \sum_{j=0}^{D_0} h_{21,i} h_{0,j} X_1(t-i-j)$$

$$+ \sum_{i=0}^{D_{21}} h_{21,i} X_{21}(t-i) + \Gamma \sum_{i=0}^{D_{21}} h_{21,i} Z_0(t-i) + Z_1(t).(10)$$

[3]Alternately, we can assume that this processing time delay is already incorporated in D_{21} and D_{22}.

TABLE I
TABLE OF $\tilde{h}_{1d,i}$, WHERE $d \in \{1,2\}$ AND $\Gamma \triangleq \dfrac{\sqrt{\alpha P_2}}{\sqrt{\sigma_0^2 + \sum_{j=0}^{D_0}|h_{0,j}|^2 P_1}}$

$D_0 \leq D_{2d} \leq D_{1d}$	$0 \leq i \leq D_0$	$D_0 + 1 \leq i \leq D_{2d}$	$D_{2d} + 1 \leq i \leq D_{1d}$
$\tilde{h}_{1d,i}$	$h_{1d,i} + \Gamma \sum_{j=0}^{i} h_{2d,i-j}h_{0,j}$	$h_{1d,i} + \Gamma \sum_{j=0}^{D_0} h_{2d,i-j}h_{0,j}$	$h_{1d,i}$
$D_{2d} \leq D_0 \leq D_{1d}$	$0 \leq i \leq D_{2d}$	$D_{2d} + 1 \leq i \leq D_0$	$D_0 + 1 \leq i \leq D_{1d}$
$\tilde{h}_{1d,i}$	$h_{1d,i} + \Gamma \sum_{j=0}^{i} h_{2d,j}h_{0,i-j}$	$h_{1d,i} + \Gamma \sum_{j=0}^{D_{2d}} h_{2d,j}h_{0,i-j}$	$h_{1d,i}$
$D_0 \leq D_{1d} \leq D_{2d}$	$0 \leq i \leq D_0$	$D_0 + 1 \leq i \leq D_{1d}$	$D_{1d} + 1 \leq i \leq D_{2d}$
$\tilde{h}_{1d,i}$	$h_{1d,i} + \Gamma \sum_{j=0}^{i} h_{2d,i-j}h_{0,j}$	$h_{1d,i} + \Gamma \sum_{j=0}^{D_0} h_{2d,i-j}h_{0,j}$	$\Gamma \sum_{j=0}^{D_0} h_{2d,i-j}h_{0,j}$
$D_{1d} \leq D_0 \leq D_{2d}$	$0 \leq i \leq D_{1d}$	$D_{1d} + 1 \leq i \leq D_0$	$D_0 + 1 \leq i \leq D_{2d}$
$\tilde{h}_{1d,i}$	$h_{1d,i} + \Gamma \sum_{j=0}^{i} h_{2d,i-j}h_{0,j}$	$\Gamma \sum_{j=0}^{i} h_{2d,i-j}h_{0,j}$	$\Gamma \sum_{j=0}^{D_0} h_{2d,i-j}h_{0,j}$
$D_{1d} \leq D_{2d} \leq D_0$	$0 \leq i \leq D_{1d}$	$D_{1d} + 1 \leq i \leq D_{2d}$	$D_{2d} + 1 \leq i \leq D_0$
$\tilde{h}_{1d,i}$	$h_{1d,i} + \Gamma \sum_{j=0}^{i} h_{2d,j}h_{0,i-j}$	$\Gamma \sum_{j=0}^{i} h_{2d,j}h_{0,i-j}$	$\Gamma \sum_{j=0}^{D_{2d}} h_{2d,j}h_{0,i-j}$
$D_{2d} \leq D_{1d} \leq D_0$	$0 \leq i \leq D_{2d}$	$D_{2d} + 1 \leq i \leq D_{1d}$	$D_{1d} + 1 \leq i \leq D_0$
$\tilde{h}_{1d,i}$	$h_{1d,i} + \Gamma \sum_{j=0}^{i} h_{2d,j}h_{0,i-j}$	$h_{1d,i} + \Gamma \sum_{j=0}^{D_{2d}} h_{2d,j}h_{0,i-j}$	$\Gamma \sum_{j=0}^{D_{2d}} h_{2d,j}h_{0,i-j}$

Letting $A \triangleq \sum_{i=0}^{D_{11}} h_{11,i}X_1(t-i) + \Gamma\sum_{i=0}^{D_{21}}\sum_{j=0}^{D_0} h_{21,i}h_{0,j}X_1(t-i-j)$ in (10), A can be rewritten, especially for $D_{11} \geq D_{21} \geq D_0$, by

$$A = \sum_{i=0}^{D_0}\left\{h_{11,i} + \Gamma\sum_{j=0}^{i}h_{21,i-j}h_{0,j}\right\}X_1(t-i)$$
$$+ \sum_{i=D_0+1}^{D_{21}}\left\{h_{11,i} + \Gamma\sum_{j=0}^{D_0}h_{21,i-j}h_{0,j}\right\}X_1(t-i)$$
$$+ \sum_{i=D_{21}+1}^{D_{11}}h_{11,i}X_1(t-i). \tag{11}$$

Therefore, (10) can be rewritten in general by

$$Y_1(t) = \sum_{i=0}^{\bar{D}_1}\tilde{h}_{11,i}X_1(t-i) + \sum_{i=0}^{D_{21}}h_{21,i}X_{21}(t-i)$$
$$+ \Gamma\sum_{i=0}^{D_{21}}h_{21,i}Z_0(t-i) + Z_1(t), \tag{12}$$

where $\bar{D}_1 \triangleq \max\{D_{11}, D_{21}, D_0\}$ and $\tilde{h}_{11,i}$'s are given in Table I. Note that comparing with (1), the signal vector $\underline{X}_1$ in (12) is more strengthen by increasing $h_{11,i}$ and D_{11} to $\tilde{h}_{11,i}$ and $\tilde{D}_{11}$, respectively, while extra interference terms such as $\sum_{i=0}^{D_{21}}h_{21,i}X_{21}(t-i) + \Gamma\sum_{i=0}^{D_{21}}h_{21,i}Z_0(t-i)$ are introduced.

Similarly, from (6) and (9), the received signal at $\mathcal{D}_2$ is given by

$$Y_2(t) = \sum_{i=0}^{D_{12}}h_{12,i}X_1(t-i) + \sum_{i=0}^{D_{22}}h_{22,i}\left\{\Gamma\sum_{j=0}^{D_0}h_{0,j}X_1(t-i-j)\right.$$
$$\left. + X_{21}(t-i) + \Gamma Z_0(t-i)\right\} + Z_2(t)$$
$$= \sum_{i=0}^{D_{12}}h_{12,i}X_1(t-i) + \Gamma\sum_{i=0}^{D_{22}}\sum_{j=0}^{D_0}h_{22,i}h_{0,j}X_1(t-i-j)$$
$$+ \sum_{i=0}^{D_{22}}h_{22,i}X_{21}(t-i) + \Gamma\sum_{i=0}^{D_{22}}h_{22,i}Z_0(t-i) + Z_2(t). \tag{13}$$

Then, (13) can be also rewritten by

$$Y_2(t) = \sum_{i=0}^{\bar{D}_2}\tilde{h}_{12,i}X_1(t-i) + \sum_{i=0}^{D_{22}}h_{22,i}X_{21}(t-i)$$
$$+ \Gamma\sum_{i=0}^{D_{22}}h_{22,i}Z_0(t-i) + Z_2(t), \tag{14}$$

where $\bar{D}_2 \triangleq \max\{D_{12}, D_{22}, D_0\}$ and $\tilde{h}_{12,i}$'s are given in Table I.

In the proposed scheme, $\mathcal{S}_2$ performs a power allocation $\Sigma_2(w)$ over frequency domain w for $\underline{X}_{21}$ with the average power constraint of $(1 - \alpha P_2)$, while $\mathcal{S}_1$ transmits data with its transmit power allocation $\Sigma_1^*(w)$ in (3) since $\mathcal{S}_1$ operates without any prior knowledge on $\mathcal{S}_2$ and $\mathcal{D}_2$. Assuming that both $\bar{D}_1$ and $\bar{D}_2$ are much smaller than the size of the FT (codeword length) T, i.e., $\bar{D}_i \ll T$, $i \in \{1,2\}$, $\mathcal{D}_1$ and $\mathcal{D}_2$ can still decompose the channel into independent parallel memoryless channels by using the FT. Then, $\mathcal{D}_2$ decodes only $\underline{X}_{21}$ treating the other terms in (14) as interference or noise, while $\mathcal{D}_1$ decodes only $\underline{X}_1$ treating the other terms in (12) as interference or noise.[4] As a result, an achievable rate for the secondary channel is given by

$$R_{2,2} =$$
$$\max_{\alpha,\Sigma_2(w)}\frac{1}{\pi}\int_0^{\pi}C\left(\frac{\Sigma_2(w)|H_{22}(w)|^2}{\sigma_2^2 + \Gamma^2|H_{22}(w)|^2\sigma_0^2 + \Sigma_1^*(w)|\tilde{H}_{12}(w)|^2}\right)dw \tag{15a}$$

subject to $0 \leq \alpha \leq 1$, (15b)

$$\Sigma_2(w) \geq 0 \text{ for all } w \tag{15c}$$

$$\frac{1}{\pi}\int_0^{\pi}\Sigma_2(w)dw \leq (1-\alpha)P_2, \tag{15d}$$

$$\sigma_1^2 + \Gamma^2|H_{21}(w)|^2\sigma_0^2 + \Sigma_2(w)|H_{21}(w)|^2 \leq \frac{|\tilde{H}_{11}(w)|^2\sigma_1^2}{|H_{11}(w)|^2}$$
$$\text{for all } w \text{ satisfying } \Sigma_1^*(w) > 0, \tag{15e}$$

where the last constraint in (15) comes from $\frac{\Sigma_1^*(w)|H_{11}(w)|^2}{\sigma_1^2} \leq \frac{\Sigma_1^*(w)|\tilde{H}_{11}(w)|^2}{\sigma_1^2 + \Gamma^2|H_{21}(w)|^2\sigma_0^2 + \Sigma_2(w)|H_{21}(w)|^2}$ so that the received SNR at

[4]When the primary channel is a flat fading channel ($D_{11} = 0$), $\mathcal{D}_1$ does not need to perform the FT. But, if there is a positive delay $\bar{D}_1 > 0$, i.e., $D_0 + D_{21} > 0$, the FT is needed for $\mathcal{D}_1$ to decode $\underline{X}_1$ from (12). Since this means that $\mathcal{D}_1$ performs an additional processing for the secondary nodes, we do not consider this case ($D_{11} = 0$).

TABLE II
TABLE OF NORMALIZED CHANNEL GAIN VECTORS

$\frac{\|\underline{h}_0\|^2}{\|\underline{h}_0\|_2^2} = (0.4766, 0.4667, 0.0215, 0.0352)$
$\frac{\|\underline{h}_{11}\|^2}{\|\underline{h}_{11}\|_2^2} = (0.5859, 0.2834, 0.0751, 0.0530, 0.0025)$
$\frac{\|\underline{h}_{12}\|^2}{\|\underline{h}_{12}\|_2^2} = (0.8144, 0.0478, 0.0870, 0.0433, 0.0059, 0.0016)$
$\frac{\|\underline{h}_{21}\|^2}{\|\underline{h}_{21}\|_2^2} = (0.8816, 0.0925, 0.0011, 0.0172, 0.0055, 0.0009, 0.0005,$ $0.0008)$
$\frac{\|\underline{h}_{22}\|^2}{\|\underline{h}_{22}\|_2^2} = (0.6981, 0.1862, 0.1124, 0.0033)$

$\mathcal{D}_1$ is kept even after $\mathcal{S}_2$ transmits data. Then, the optimal power allocation $\Sigma_2^*(w)$ is given by

$$\Sigma_2^*(w) = \begin{cases} \min\left\{ \left[\frac{1}{\lambda_{2,2} \ln 2} - \frac{\sigma_2^2 + \Sigma_1^*(w)|\tilde{H}_{12}(w)|^2}{|H_{22}(w)|^2} - \Gamma^2 \sigma_0^2 \right]^+, \right. \\ \left. \left[\frac{\sigma_1^2}{|H_{21}(w)|^2} \left(\frac{|\tilde{H}_{11}(w)|^2}{|H_{11}(w)|^2} - 1 \right) - \Gamma^2 \sigma_0^2 \right]^+ \right\}, \text{for } \Sigma_1^*(w) > 0 \\ \left[\frac{1}{\lambda_{2,2} \ln 2} - \frac{\sigma_2^2}{|H_{22}(w)|^2} - \Gamma^2 \sigma_0^2 \right]^+, \text{ for } \Sigma_1^*(w) = 0, \end{cases}$$
(16)

where $\lambda_{2,2}$ denotes Lagrange multiplier associated with the constraint (15d). It is notable that if $\alpha = 0$, then the proposed scheme is the same as the conventional scheme, and as a result (15) and (16) are reduced to (7) and (8), respectively.

IV. NUMERICAL EXAMPLES

In this section, we provide some numerical examples of the conventional and the proposed CR schemes through computer simulation. The simulation environments are as follows. The channel tap values for $\underline{h}_0$, $\underline{h}_{11}$, $\underline{h}_{12}$, $\underline{h}_{21}$, and $\underline{h}_{22}$ are chosen to make their normalized gains be same as shown in Table II, where we assume that the interfering ISI channels such as $\underline{h}_{12}$ and $\underline{h}_{21}$ have larger delay spreads than those of the other ISI channels such that $D_0 = 3$, $D_{11} = 4$, $D_{12} = 5$, $D_{21} = 7$, and $D_{22} = 3$. After defining three different SNRs such as $\text{SNR}_0 \triangleq \frac{P_1 \|\underline{h}_0\|_2^2}{\sigma_0^2}$, $\text{SNR}_1 \triangleq \frac{P_1 \|\underline{h}_{11}\|_2^2}{\sigma_1^2} = \frac{P_2 \|\underline{h}_{21}\|_2^2}{\sigma_1^2}$, and $\text{SNR}_2 \triangleq \frac{P_1 \|\underline{h}_{12}\|_2^2}{\sigma_2^2} = \frac{P_2 \|\underline{h}_{22}\|_2^2}{\sigma_2^2}$, we then obtain simulation results by varying one SNR value while the other two SNR values are fixed.

In Fig. 2, C_1 represents primary channel capacity. $R_{2,1}$ and $R_{2,2}$ represent the achievable rates for secondary channel of the conventional and proposed schemes, respectively. We first notify that $R_{2,2}$ always outperforms $R_{2,1}$ as shown in Fig. 2. Fig. 2 (a) shows that $R_{2,2}$ increases as SNR_0 increases. This is because when the proposed CR scheme amplifies the received signal $\underline{Y}_0$ at $\mathcal{S}_2$ to construct $\underline{X}_2$, the noise term $\underline{Z}_0$ in $\underline{Y}_0$ is also amplified. Since the conventional scheme does not utilize $\underline{Y}_0$, $R_{2,1}$ does not change for varying SNR_0 values. Fig. 2 (b) shows that both $R_{2,1}$ and $R_{2,2}$ decrease as SNR_1 increases. As the primary nodes operate in higher SNR_1 values, less frequency bands are left unused by the primary nodes, and hence $R_{2,1}$ decreases. Moreover, as SNR_1 increases, we found that α^* in the proposed scheme is chosen to larger values, and hence $R_{2,2}$ also decreases. On the other hand, Fig. 2 (c) shows that both $R_{2,1}$ and $R_{2,2}$ increase as SNR_2 increases when $\text{SNR}_0 = 10$ [dB] and $\text{SNR}_1 = 3$ [dB]. This is because the primary nodes operate in a relatively low SNR value, and
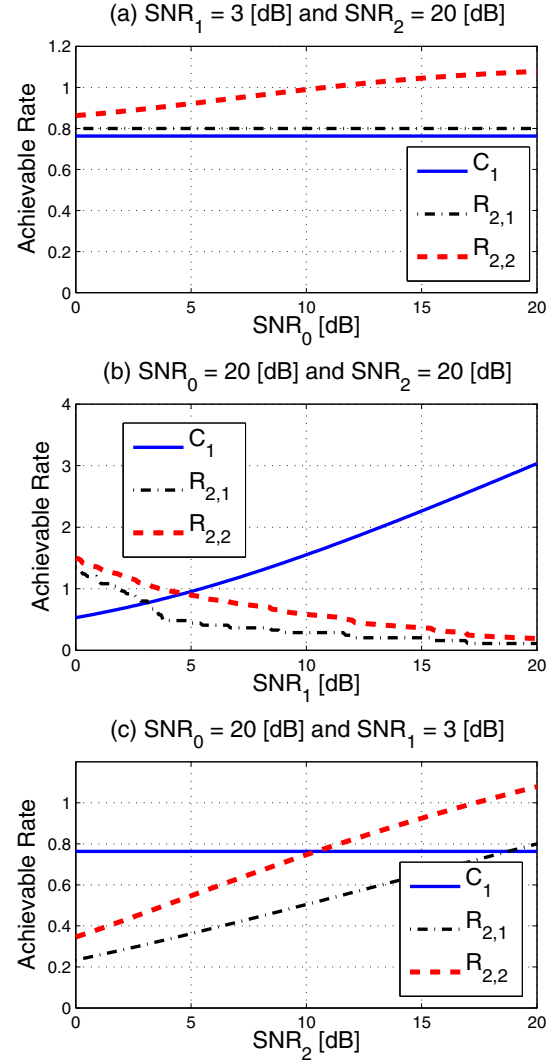

Fig. 2. C_1, $R_{2,1}$ and $R_{2,2}$ versus SNR_i. (a) $i = 0$, $\text{SNR}_1 = 3$ [dB], and $\text{SNR}_2 = 20$ [dB], (b) $i = 1$, $\text{SNR}_1 = 3$ [dB], and $\text{SNR}_2 = 20$ [dB], and (c) $i = 2$, $\text{SNR}_0 = 10$ [dB], and $\text{SNR}_1 = 3$ [dB].

hence the secondary nodes can fully utilize enough number of frequency bands which are not used by the primary nodes.

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